FORECASTING IN THE TROPICS WITH A BAROTROPIC ATMOSPHERIC MODEL

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ABSTRACT

A series of barotropic forecasts has been prepared for several upper-air levels in the tropical Pacific. The governing equation is the vorticity equation for a barotropic non-divergent atmosphere. The input data are stream-function values derived from an objective tropical wind analysis. The area covered is the Pacific Ocean Tropics of both the Northern and Southern Hemispheres. Examples of tropical barotropic forecasts are shown. Verification data suggest that these barotropic forecasts have slightly less accuracy than subjective forecasts.

1. INTRODUCTION

The first attempt to apply the vorticity equation for a barotropic atmosphere to numerical prediction of upper-air flow patterns in the Tropics was that of Jordan [7]. The next attempt was that of Rosenthal [8]. The latter author mentions unpublished work in this field by J. B. Knox and by D. F. Rex and R. A. Brownlee.

During the first part of 1963 a U.S. Weather Bureau and Air Force group of meteorologists succeeded in making satisfactory objective (computer) wind analyses for the tropical Pacific areas of both the Northern and Southern Hemispheres for several upper-air levels [1]. Their next step]2] was to make statistical forecasts from these analyses. Their latest step is the preparation of tropical barotropic forecasts, some of which will be discussed in this paper. It should be considered a progress report on the joint efforts of the Weather Bureau, University of Hawaii, Air Force, and Navy in Honolulu to apply numerical and dynamical methods to forecasting in the Tropics.

2. GOVERNING EQUATION

For a non-divergent barotropic atmosphere the governing equation is that of conservation of absolute vorticity

$$\frac{\partial}{\partial t}\nabla^2\psi+J(\psi,\eta)=0 \tag{1}$$

where ψ is the stream function, J is the Jacobian operator, and η is the absolute vorticity.

In finite-difference form equation (1) may be written

$$\frac{\partial}{\partial t}\nabla^2\psi+\frac{1}{4}J(\psi,\eta)=0 \tag{2}$$

where

$$\nabla^2\psi=\psi_{i+1,j}+\psi_{i-1,j}+\psi_{i,j+1}+\psi_{i,j-1}-4\psi_{i,j}$$

and

$$J(\psi,\eta)=[(\psi_{i+1,j}-\psi_{i-1,j})(\eta_{i,j+1}-\eta_{i,j-1}) -(\psi_{i,j+1}-\psi_{i,j-1})(\eta_{i+1,j}-\eta_{i-1,j})].$$

The subscripts refer to grid-point locations.

The time derivative in the first term of equation (2) may also be expressed in finite-difference form:

$$\frac{\partial}{\partial t}\nabla^2\psi=\frac{\nabla^2\psi_{t+\Delta t}-\nabla^2\psi_{t-\Delta t}}{2\Delta t}. \tag{3}$$

With the aid of equation (3) we may re-write equation (2) as

$$\begin{aligned}\nabla^2\psi_{t+\Delta t}=&(\psi_{i+1,j}+\psi_{i-1,j}+\psi_{i,j+1}+\psi_{i,j-1}-4\psi_{i,j})_{t-\Delta t}\\&-\frac{\Delta t}{2}[(\psi_{i+1,j}-\psi_{i-1,j})(\eta_{i,j+1}-\eta_{i,j-1})\\&-(\psi_{i,j+1}-\psi_{i,j-1})(\eta_{i+1,j}-\eta_{i-1,j})]_t\end{aligned} \tag{4}$$

which is the governing equation for the non-divergent barotropic model in finite difference form.

In solving equation (4), ψ was kept constant along the boundary during the forecast period ($\partial\psi/\partial t=0$), and the absolute vorticity along the boundary was taken equal to f, the Coriolis parameter ($\eta=f$). To eliminate difficulties connected with the breakdown of the geostrophic relationship at the equator, the reported heights of constant-pressure surfaces were not used in the analyses and forecasts. We avoided any assumption of the relationship of the height field to the wind field whether geostrophic, gradient, or "balance." In this connection it may be interesting to note that since equation (4) involves only the wind field and the Coriolis parameter there is no intrinsic requirement for use of the height field.

From the routinely prepared computer wind analyses we obtained the stream-function fields [2] and this enabled us to compute the fields of absolute vorticity required for the solution of equation (4). A relaxation technique gave the value of ψ at the time $t+\Delta t$; Δt was taken to be 1 hour.

From a consideration of the scales of atmospheric motion, Charney [3] concluded that, in the absence of condensation, tropical synoptic scale motions are quasi-horizontal and quasi-non-divergent and that equation (1) is a good approximation to the law governing synoptic-scale circulation in the Tropics.

3. COMPUTER PROGRAM

Our computer analysis and forecast area extended from 37°N. to 24°S. and from 110°W. westward across the Pacific to 100°E. The grid length was 5° longitude at the equator. A network of 30×14 grid points on a Mercator map projection covered the area, which included most of the Pacific Ocean Tropics of both the Northern and Southern Hemispheres. The forecasts were made on the IBM 7040 computer of the computing center of the University of Hawaii.

The barotropic program was written in FORTRAN language and contains about 200 statements. It is based on a program originally written in the Joint Numerical Weather Prediction Unit at Suitland, Md., and later revised for use by the National Hurricane Research Project.

Barotropic 24-hr. forecasts were made for the 700-mb., 500-mb., 300-mb., and 200-mb. levels for 35 cases. Forecasts of wind, stream function, and absolute vorticity were printed by the computer. It took about 12 min. to produce the forecasts for the four levels.

It would seem that in the real atmosphere the non-divergent barotropic equation (4) would apply best at the equivalent barotropic level. But the literature gives no indication of where this level is in the Tropics. We, therefore, decided to apply equation (4) at each of the four levels for which an objective analysis of the wind field was available with the hope that forecasts for one of the levels would turn out to be much superior to those for the others. A similar technique was employed by Cressman [5] to estimate the height of the equivalent barotropic level in mid-latitudes.

4. EXAMPLES OF TROPICAL BAROTROPIC FORECASTS

Figures 1 to 4 show the initial streamline and absolute vorticity analyses, the 24-hr. barotropic forecasts, and the verifying charts. Initial time was 0000 GMT, January 4, 1965, and the forecasts verified at 0000 GMT, January 5, 1965. The analyses as well as the forecasts were computer-produced.

In evaluating the quality of the forecasts, one should keep in mind that stream-function values are fixed along the boundaries throughout the forecast period. Synoptic systems cannot move along the boundaries nor enter from the outside. Synoptic systems from the interior arriving at a boundary will tend to become distorted.

Turning our attention to the Northern Hemisphere portion of the 700-mb. level (fig. 1), we notice that the clockwise circulation in the western Pacific was predicted fairly well; and so was the one near Honolulu. The location of the former circulation center is not so clear in figure 1c but when "intermediate" isolines, smaller stream-function intervals, are drawn it shows up in the western lobe of the large closed clockwise circulation.

The southern parts of the troughs near 175° W. and near the United States coast were moved too slowly in the forecast. This, together with the maintenance of fixed stream-function values along the boundaries, has lead to an erroneous NE to SW tilt of the troughs. There are several drastic assumptions implicit in our application of equation (1) to the real atmosphere: we have neglected the effects of divergence, latent and sensible heat, friction, etc. To establish the reason for the errors in our forecasts by determining the quantitative effect of each assumption would require further experimentation with more sophisticated atmospheric models.

Since our basic equation is that expressing the conservation of absolute vorticity it may be interesting to examine the forecast with this in mind. The maximum value of 80 ($=80\times10^{-6}$ sec.$^{-1}$) is observed on January 4 in the troughs near the northern boundary of the chart; and this is the maximum value on the forecast chart. No new values of absolute vorticity have been introduced into the forecast by the finite difference approximations, and existing values have been carried along. In the Southern Hemisphere the forecast vorticity isolines on the initial and forecast charts are about the same because advection is very weak.

It would be reasonable to ask how this 700-mb. computer forecast compared with the one made by the human forecaster. For several years we have had a station forecast verification program which consists in computing the average magnitude of the vector differences between forecast and observed winds at 20 fixed stations scattered over the Pacific. Application of this system to the barotropic forecast gave an average error of 13 kt.; the official forecaster made an error of 19 kt.

It has been suggested that it might be better to compare the barotropic forecasts with persistence forecasts or with persistence-climatology forecasts. Tests over a 4-year period have shown that the official Honolulu forecaster, on the average, makes better forecasts than forecasts produced by either of those methods. The barotropic forecasts are, therefore, being compared here with our "best" forecasts.

A "stream-wind scale" has been included in figure 1a to enable one to estimate the wind speed from the stream-function gradient. It is to be used like the ordinary geostropic wind scale. This follows from the relationship $\mathbf{V}=\mathbf{k}\times\nabla\Psi$, where $\mathbf{V}$ is the vector wind velocity and $\mathbf{k}$ the unit vertical vector. The variation with latitude of the map-scale factor is the reason the stream-function spacing, for a given wind speed, increases with map distance from the equator.

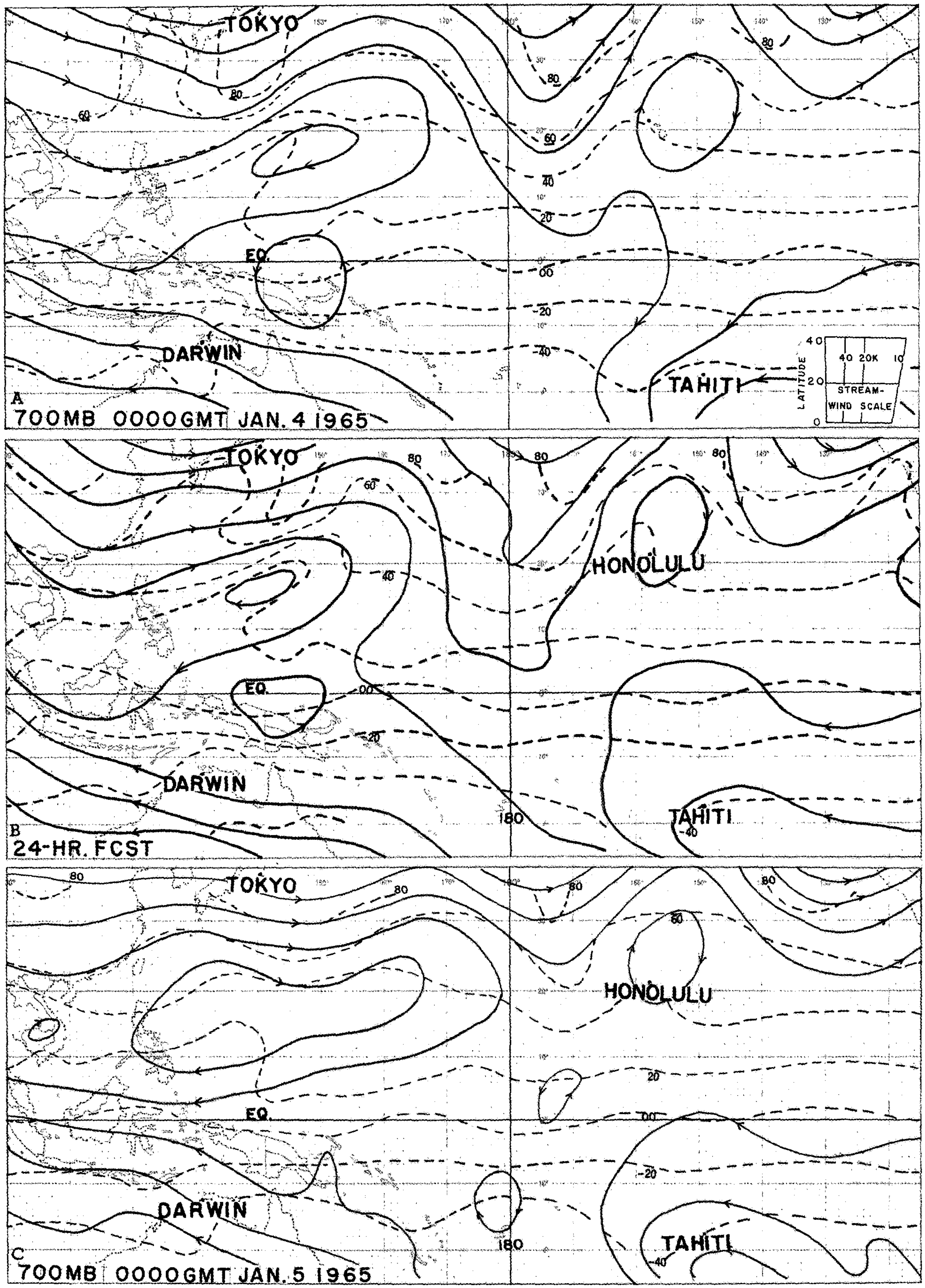

FIGURE 1.—(a) Stream-function (solid lines) and absolute vorticity (dashed lines) fields, 700 mb., 0000 GMT January 4, 1965. Absolute vorticity units 10^{-6} sec.$^{-1}$ (b) The 24-hr. barotropic forecasts of the 700-mb. stream-function and absolute vorticity fields verifying at 0000 GMT January 5, 1965. (c) The observed 700-mb. stream-function and absolute vorticity fields, 0000 GMT January 5, 1965.

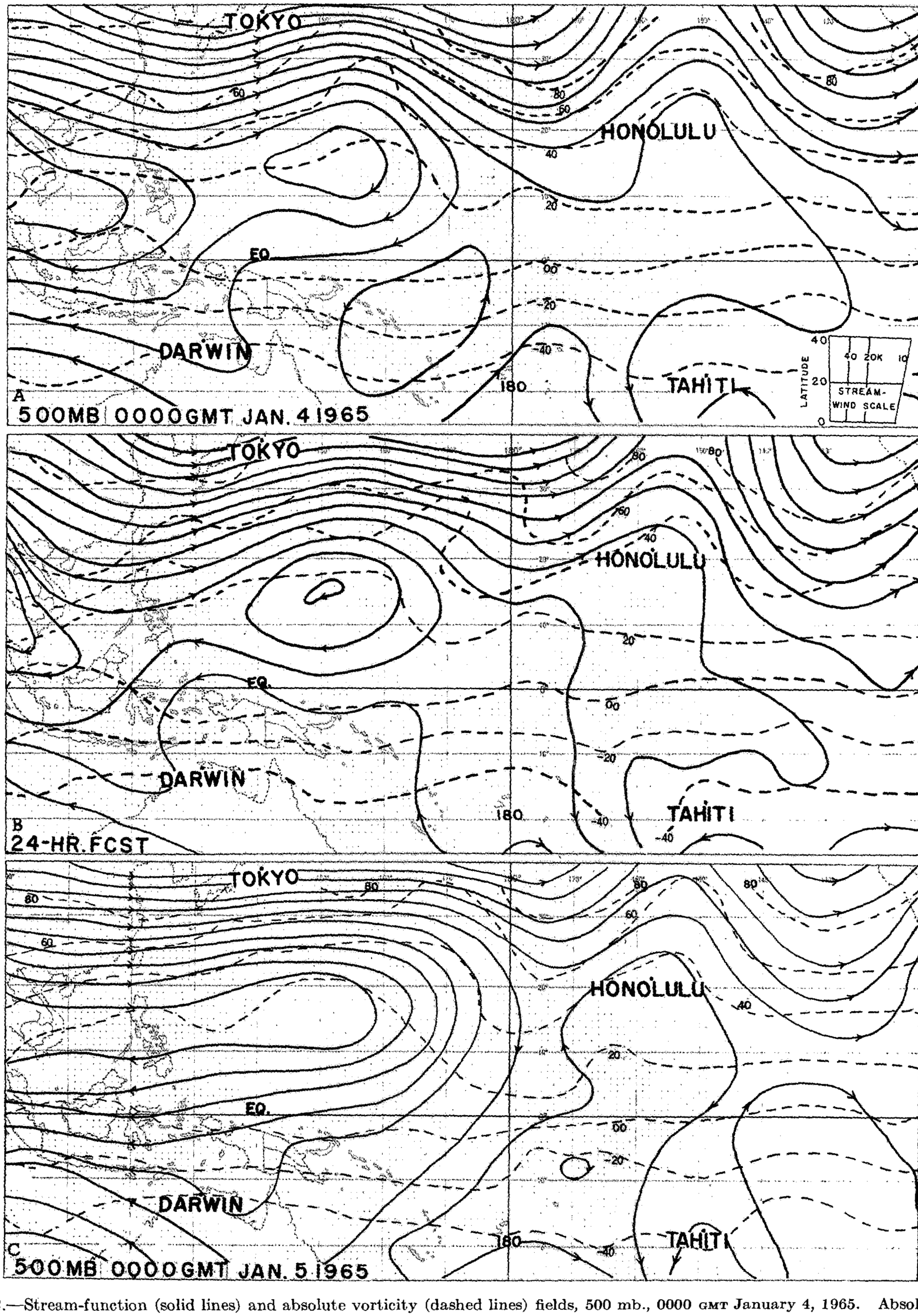

FIGURE 2.—Stream-function (solid lines) and absolute vorticity (dashed lines) fields, 500 mb., 0000 GMT January 4, 1965. Absolute vorticity units 10^{-6} sec.$^{-1}$ (b) The 24-hr. barotropic forecasts of the 500-mb. stream-function and absolute vorticity fields verifying at 0000 GMT January 5, 1965. (c) The observed 500-mb. stream-function and absolute vorticity fields, 0000 GMT January 5, 1965.

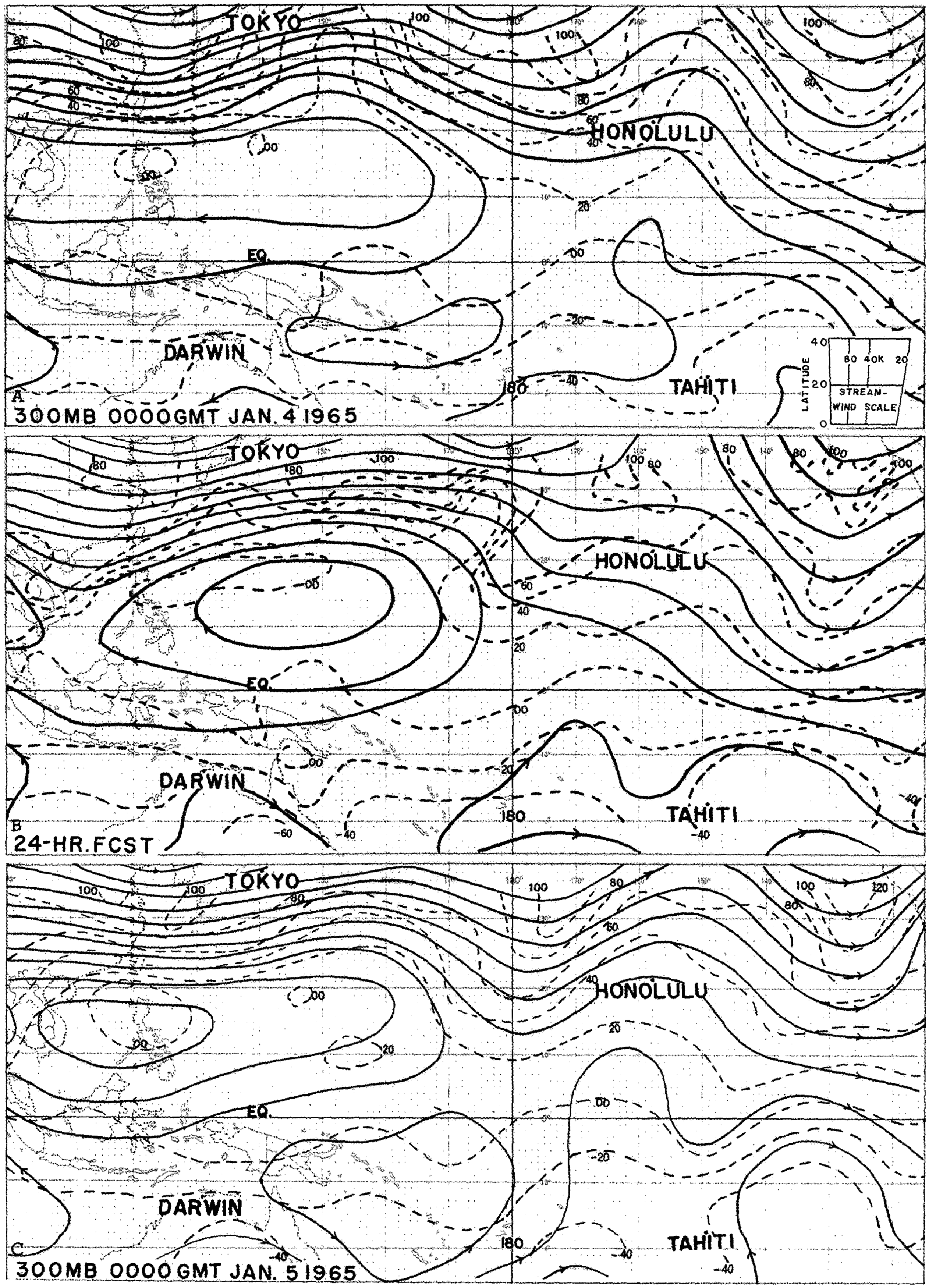

FIGURE 3.—Stream-function (solid lines) and absolute vorticity (dashed lines) fields, 300 mb., 0000 GMT January 4, 1965. Absolute vorticity units 10^{-6} sec.$^{-1}$ (b) The 24-hr. barotropic forecasts of the 300-mb. stream-function and absolute vorticity fields verifying at 0000 GMT January 5, 1965. (c) The observed 300-mb. stream-function and absolute vorticity fields, 0000 GMT January 5, 1965.

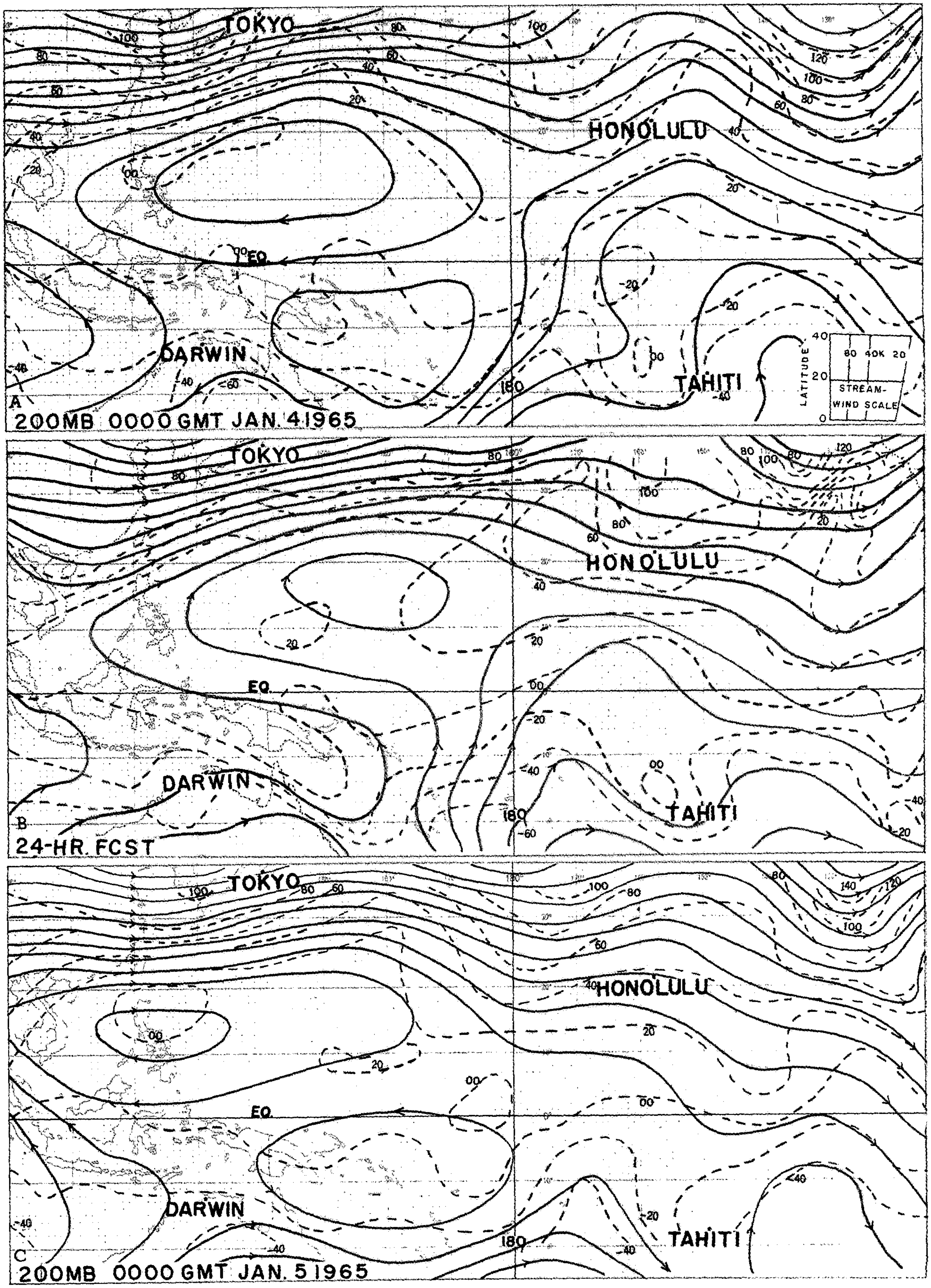

FIGURE 4.—(a) Stream-function (solid lines) and absolute vorticity (dashed lines) fields, 200 mb., 0000 GMT January 4, 1965. Absolute vorticity units 10^{-6} sec.$^{-1}$ (b) The 24-hr. barotropic forecasts of the 200-mb. stream-function and absolute vorticity fields verifying at 0000 GMT January 5, 1965. (c) The observed 200-mb. stream-function and absolute vorticity fields, 0000 GMT January 5, 1965.

At 500 mb. (fig. 2) the two most pronounced ridge lines, the one in the western North Pacific stretching approximately E-W from 10° to 15° N. and the one extending N-S near Honolulu, have, again, been well predicted. The predicted maintenance of a weaker ridge near 155° E. failed.

Of the three major troughs in the westerlies, those in the eastern Pacific near 175° W. and 130° W. verified well. But the trough off the coast of Asia is less pronounced than predicted. Tentatively, we ascribe this failure to developments over the Asian continent beyond the western boundary.

The absolute vorticity patterns, predicted and observed, have much in common. The barotropic 500-mb. forecast had an error of 13 kt.; the official forecaster an error of 16 kt.

At the higher levels the barotropic forecasts are not as good as the 500-mb. forecast. The 300-mb. ridge line (fig.3) in the western Pacific stretching E-W along, roughly, 15° N. has been well placed in the forecast but the predicted clockwise circulation center is much too far east. The simplest tentative explanation of the error seems to be the artificial boundary condition, which prevents synoptic developments over Asia from penetrating the forecast area.

The trough-ridge pattern between the date-line and Honolulu has been erroneously suppressed in the forecast. A proposed explanation is that the system is not nondivergent but has a convergence-divergence mechanism operating to sharpen the trough-ridge pattern more than implied by equation (1).

The forecast for the trough in the eastern Pacific was fairly good even though the slight NE-SW tilt failed to develop.

The weak summer vorticity advection patterns in the Southern Hemisphere make for little change in the predicted and observed circulation patterns. The barotropic 300-mb. forecast had an error of 25 kt.; the official forecaster an error of 22 kt.

At the highest level, 200 mb. (fig. 4), the forecast for the E-W ridge line in the western North Pacific turned out fairly well but the associated clockwise circulation center has the same kind of error as just noted at 300 mb.—and probably for the same reason.

Again, as at 300 mb., the trough-ridge pattern between the date-line and Honolulu has been treated poorly—and probably for the same reason as at 300 mb.

In the Southern Hemisphere the observed circulation pattern differs considerably from that of the barotropic forecast. While part of the difference may be ascribed to the faulty atmospheric model, in this case it seems more likely to be the result of analysis vagaries connected with a skimpy number of observations at 200 mb.

The barotropic 200-mb. forecast had an error of 22 kt.; the official forecaster an error of 24 kt.

The verification record for all 35 barotropic forecasts for all four levels is summarized in figure 5. The main point it shows is that the official, subjective forecasts had a smaller error at all levels.

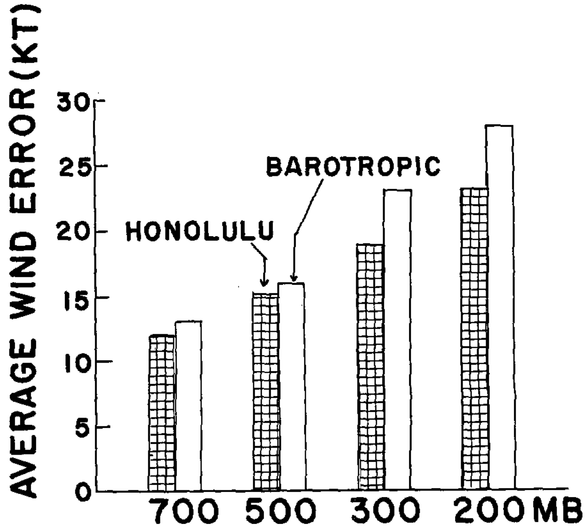

FIGURE 5.—Verification scores for official Honolulu (shaded) and barotropic forecasts (unshaded) for several levels. Each level has 35 cases. The bars indicate the average magnitude of vector differences between forecast and observed winds at selected stations.

5. DISCUSSION

It seems to be generally agreed that the equivalent barotropic level (that level at which the observed wind is about 25 percent greater than the pressure-averaged wind) in mid-latitudes is in the neighborhood of the 500-mb. level [4]. Equation (4) should apply best at the equivalent barotropic level. Since we know of no study to determine where this level is in the Tropics, we applied equation (4) at several levels. We are, however, examining the upper-air climatology to determine the average height of the equivalent barotropic level in the Tropics. Preliminary results seem to indicate that the level at which the wind in the tropical Pacific equals the pressure-averaged wind is near 500 mb.

When barotropic forecasts were first made in mid- and high-latitudes of the Northern Hemisphere several characteristic errors showed up. Among these were: (1) spurious anticyclogenesis; (2) swift retrogression of ultralong waves; (3) errors in prediction of the latitude of the maximum zonal winds. Too few tropical barotropic forecasts are available to be able to draw sound conclusions about characteristic errors, but some remarks may be of interest. Shuman [10] showed that most of the barotropic errors associated with spurious anticyclogenesis were due to the inconsistent use of the geostrophic assumption. After the balance equation replaced the geostrophic equation anticyclogenesis errors were greatly diminished. Because we have made no use of a height-wind relationship, spurious anticyclogenesis in our tropical barotropic forecasts, if it occurs, must have some other cause.

Wolff [12] re-emphasized that ultra-long Rossby waves could have remarkably large retrograde speeds. For example, at latitude 15° wave number 3 would have a westward (retrograde) speed of 68 m. sec.$^{-1}$ if the basic zonal west wind speed were 25 m. sec.$^{-1}$. A trough of wave 3 situated near the eastern boundary of our area might, under these conditions, be expected to retrograde to the mid-Pacific in 24 hr. But in actual fact ultra-long waves move very slowly. Cressman [6] found that Rossby had pointed out earlier that the contradiction could be eliminated by introducing a divergence term into equation (1). Since we have deliberately avoided at this stage, the introduction of a divergence term it seems that we have to expect ultra-long wave retrogression to spoil some of our forecasts. Experiments are now being made to determine if a divergence correction term will improve our forecasts.

Errors in the barotropic prediction of mid-latitude zonal winds were such that the speeds were too high in some latitudes and too low farther north and farther south. The introduction of a divergence term into equation (1) tends to reduce these errors [11]. No tests have yet been made to see if our tropical barotropic model has systematic errors in the prediction of the strength and location of the zonal winds.

Geographical boundaries have always created difficulties in mid-latitude prediction. It is already clear that the boundaries of our tropical area will also introduce errors into the forecast. To eliminate the errors resulting from maintaining the stream function fixed during the forecast period along the northern boundary we plan to introduce the forecast values of the stream function derived from mid-latitude models. The Australian Meteorological Service is about to tackle the problem of numerical weather prediction for part of the Southern Hemisphere. We may be able to use their results for our southern boundary.

The only way to eliminate the eastern and western boundaries is to make analyses for the Tropics of the entire globe. We have begun to examine the problems associated with preparing tropical global analyses on a daily basis and speedily enough to be used operationally [9]. It may be possible today, by collecting data from only five key offices, to get enough surface and upper-air information and rapidly enough to enable a global tropical analysis and forecast center to operate on a real time basis. The main bottleneck appears to be inefficient or non-existent communication facilities.

Our evaluation of the results of this experiment in barotropic forecasting is that it showed that (1) use of the theorem of conservation of absolute vorticity leads to realistic, but not excellent, forecasts in the Tropics; (2) considerably more experimentation has to be done with barotropic atmospheric models before they can raise the present level of tropical forecasting skill.

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